***SOLUTION Section* 3.5 – The Ratio and Root Tests**

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***











Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***











Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 









Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 









Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 



Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 









Therefore; the given series ***converges*** ***Conditionally*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use the Ratio Test to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***diverges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** ***absolutely*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** *absolutely* by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by the Root Test.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***diverges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***



Let 













Therefore; the given series ***diverges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***



Let 











Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the Root Test to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***converges*** by the Ratio Test.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

 







Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

 







Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

 









Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Using comparison method:



Since 

Therefore; the given series ***diverges*** by the *Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

For 



Therefore; the given series ***converges*** by *Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

The given series converges by comparison with 

Since 

Therefore; the given series ***converges*** by *Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Since 

Therefore; the given series ***diverges*** by *Comparison Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



 converges geometric since 







Therefore; the given series ***converges*** by *Comparison Test* with geometric series

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***diverges*** by the divergence series.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Let 







Therefore; the given series ***diverges*** to infinity by *Comparison Test* with divergent ***p****-series*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** to infinity.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by *Integral Test*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***diverges*** by *Integral Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***diverges*** to infinity by *p-*series 

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by *Comparison Test* with the Harmonic

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***diverges*** by *harmonic series*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***converges*** by ***p-****series* 

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***diverges*** by *Geometric series*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***diverges*** by n*th*-Term Test.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***diverges*** by *harmonic series*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





Therefore; the given series ***diverges*** by *Geometric series*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***converges*** by ***p-****series* 

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





 *converges*

Therefore; the given series ***converges*** by Limit Comparison Test with Geometric series 

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***diverges*** by nth-Term Test.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***converges*** by *Direct Comparison Test* with Geometric series 

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

 





Therefore; the given series ***diverges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Therefore; the given series ***converges*** by *Comparison Test* with ***p-****series* 

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Let 









Therefore; the given series ***converges*** by *Integral Test*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by *Root Test*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







 converges by ***p-****series* 

Therefore; the given series also ***converges*** by *Comparison Test*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Let 













Therefore; the given series also ***diverges*** by *Limit* *Comparison Test*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***









Therefore; the given series ***diverges*** by *Integral Test*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

 ***Telescopic series***









Therefore; the given series ***diverges***.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***





 converges by ***p-****series* 

Therefore; the given series also ***converges*** by *Comparison Test.*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

 For large *k*.





 converges by ***p-****series* 

Therefore; the given series also ***converges*** by *Comparison Test*

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***

Let 

 









 





Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***



Let 

 







Therefore; the given series ***converges*** by the *Ratio Test*.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the Ratio Test.

***Exercise***

Use any method to determine if the series converges or diverges. 

***Solution***







Therefore; the given series ***converges*** by the Ratio Test.

***Exercise***

Use the integral test to show that  converges. Show that the sum *s* of the series is less than 

***Solution***







Therefore; the given series ***converges*** by the *Integral Test* and its sum is less than 

***Exercise***

Use the root test to show that  converges

***Solution***







Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Use the root test to test that  converges

***Solution***







Therefore; the given series ***converges*** by the *Root Test*.

***Exercise***

Try to use the ratio test to determine whether  converges. What happen?

Now observe that 



Does the given series converges? Why or why not?

***Solution***







Therefore; the ratio test provides no information

However from the given:





Therefore; the given series ***diverges*** to infinity.

***Exercise***

Suppose  and  for all *n*. Show that  diverges.



***Solution***

If  and  for all *n*.

Then, by using induction







 



Therefore; the given series ***diverges*** by comparison with the harmonic series 

***Exercise***

Working in the early 1600s, the mathematicians Wallis, Pascal, and Fermat were calculating the area of the region under the curve  between  and , where *p* is the positive integer. Using arguments that predated the Fundamental Theorem of Calculus, they were able to prove that



Use Riemann sums and integrals to verify this limit.

***Solution***

The sum on the left is simply the left Riemann sum over *n* equal intervals between 0 and 1 for .

The limit of the sum is:



 

***Exercise***

Complete the following steps to find the values of  for which the series  converges

1. Use the Ratio Test to show that  converges for .
2. Use Stirling’s formula,  for large *k*, to determine whether the series converges when .



***Solution***

1. Using the Ratio Test









Therefore; the given series converges for .

1. When 

***Given***: 





***Given***: 











 

Therefore; the given series diverges for  by the *Limit Comparison Test* with ***p****-*series 